Devil is in the Details

Brian Crowder, P.E.

Food for Thought

- **Does the alternate load path analysis using “linear-elastic” techniques provide an accurate prediction of the response of the structure under consideration?**

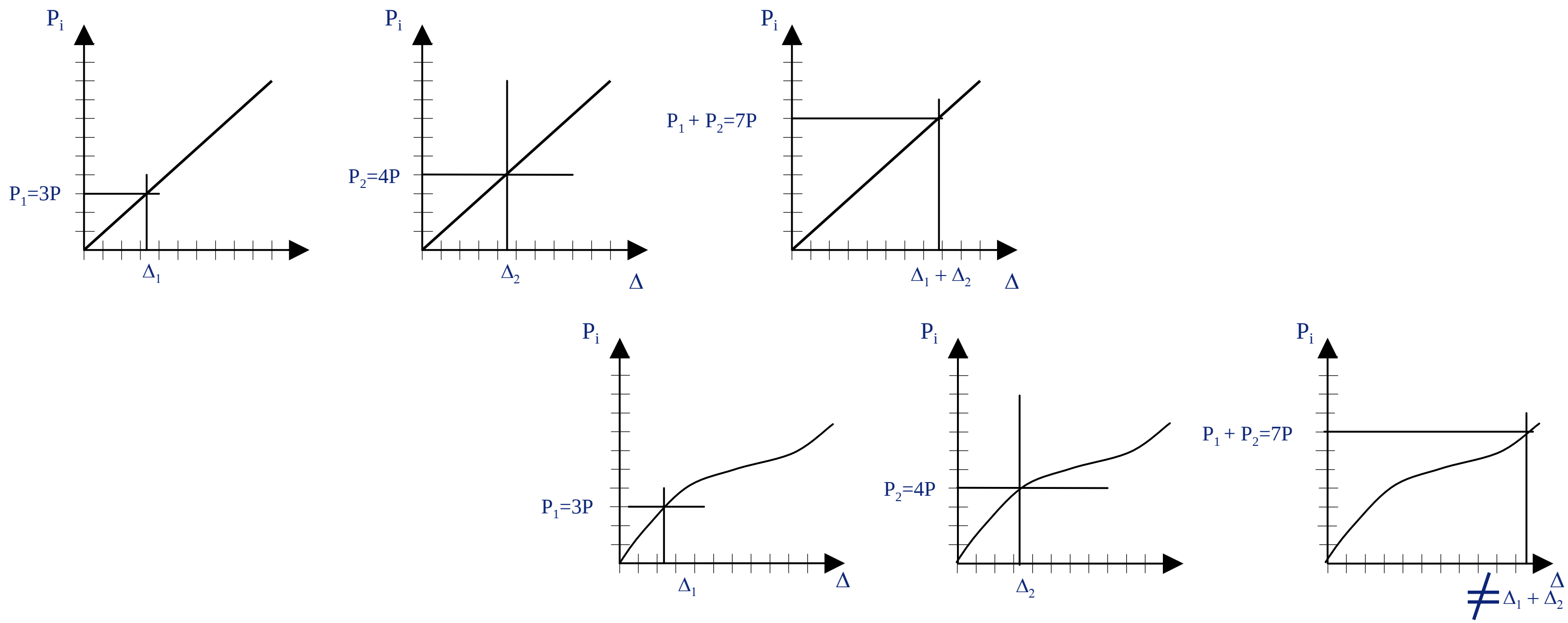
Although, hinges are incorporated into the analysis, making it really a hinge-by-hinge method, there are several factors that will impact the accuracy if the structure has non-linear action.

Food for Thought

- Linear-elastic, first order analysis is based on small deflection theory. The response limits established in the ALP method would be large deflections for most situations
- The analysis conservatively approximates dynamic or inertial effects
- The analysis procedure assumes “elastic-perfectly plastic” behavior
- Analysis won’t consider membrane response
- Second-Order Effects

Food for Thought

- **The analysis frequently assumes superposition of load effects. Is superposition valid if the response is non-linear?**



- **You must also eliminate all other limit states in order to ensure the flexural response modeled in the ALP method governs**

Food for Thought

- So it is more correct to consider a “linear-elastic” ALP analysis to be a predictor of the potential for progressive collapse in a given structure, than a predictor of the actual response.
- Properly calibrated, there isn’t anything necessarily wrong with this fact and is not dissimilar to the provision of equivalent linear elastic procedures for seismic design

Reminders for designers.....

- Designer must ensure structure is detailed to act as it is modeled in the analysis software
- Alternate Load Path analysis may assume the development of plastic hinges
- For hinges to develop, other limit states can not control the response
- What other limit states do you have to consider?

Limit States

- **Shear**
  - **Must ensure members and connections do not fail in shear before failure in flexure**
- **Local Buckling**
  - **Must ensure local buckling of slender elements doesn't occur prior to flexural response**
- **Global Buckling (Stability)**
  - **Compression elements must not fail from buckling before flexural response**
- **Lateral-Torsional Buckling**
  - **Although normally considered in calculation of flexural capacity, this type of buckling could control over development of a plastic hinge**
  - **So if you are look at value of M_n, you must know it is pure flexural limit not the moment value of lateral-torsional buckling**

Limit States

- **Connections**
  - –Connections must be stronger than element in order to ensure hinge formation in member
  - –Connections may need to accommodate large rotations, thus requiring significant ductility (no brittle failure)
  - –Must have adequate reserve of ductility after the plastic moment value has been reached, so that subsequent hinges can form throughout the structure
- **Bracing**
  - –Stability will be decreased in vicinity of hinge formation, therefore the region must be adequately braced

Effects of other loads on Mp

- Shear can influence magnitude of Mp
  - Mostly in short beams or beams with concentrated loads near the supports
  - In many cases strain hardening can counter effect
- Axial forces can reduce magnitude of Mp
  - Effect most pronounced in multi-story structures where axial load is more significant relative to member capacity
  - Axial – Moment interaction can be governed by stability or yielding and hinge formation should only be considered where yielding governs

Axial Load Effect on Mp

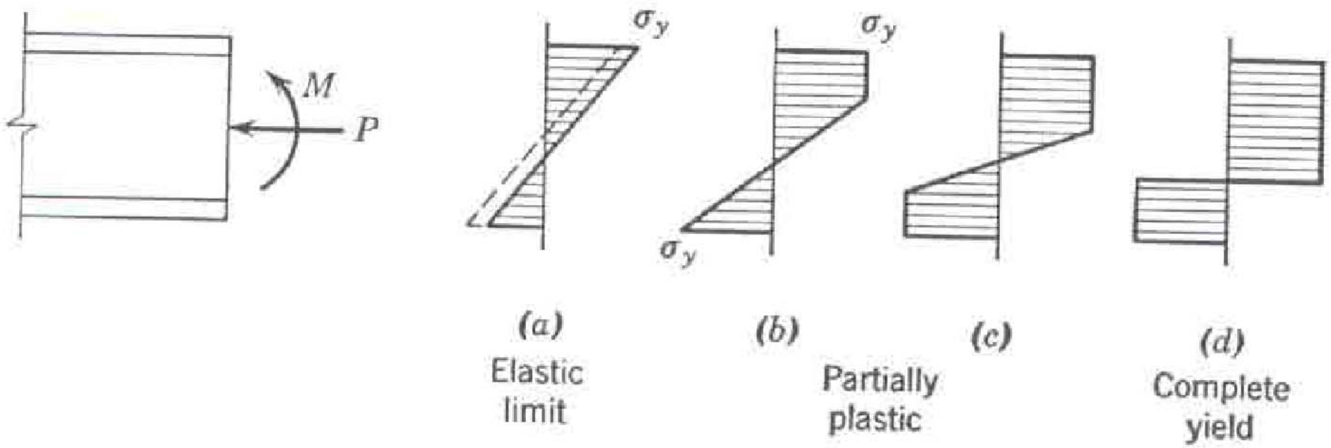
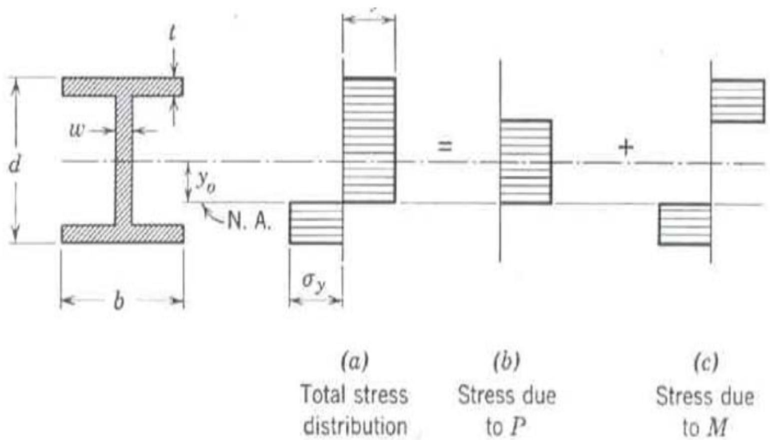


Fig. 4.1. Distribution of stress at various stages of yielding for a member subjected to bending and axial force.



Example of Axial Effect on Mp

- **When slenderness doesn't govern**

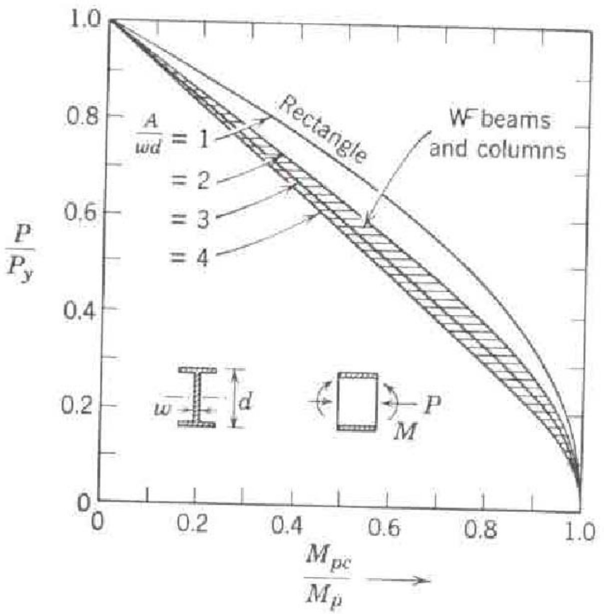


Fig. 4.4. Nondimensional interaction curve for various WF shapes.

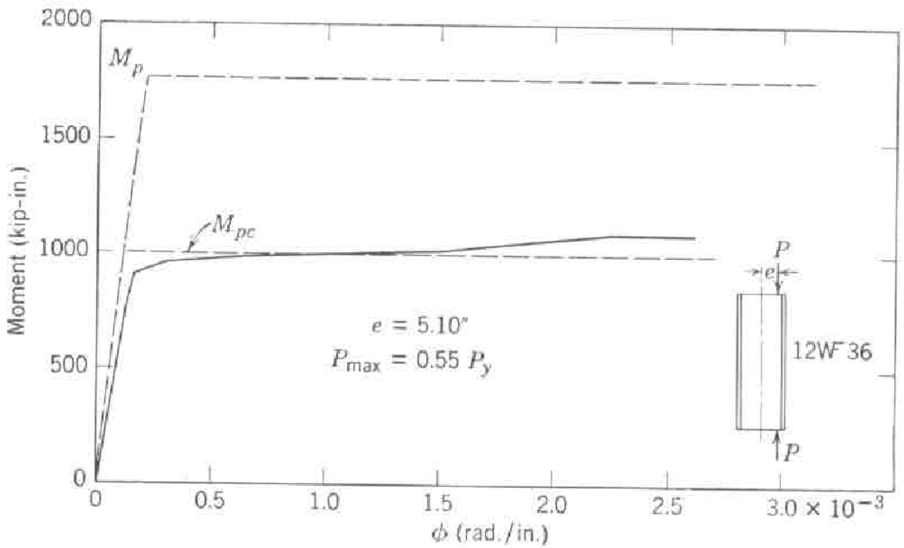
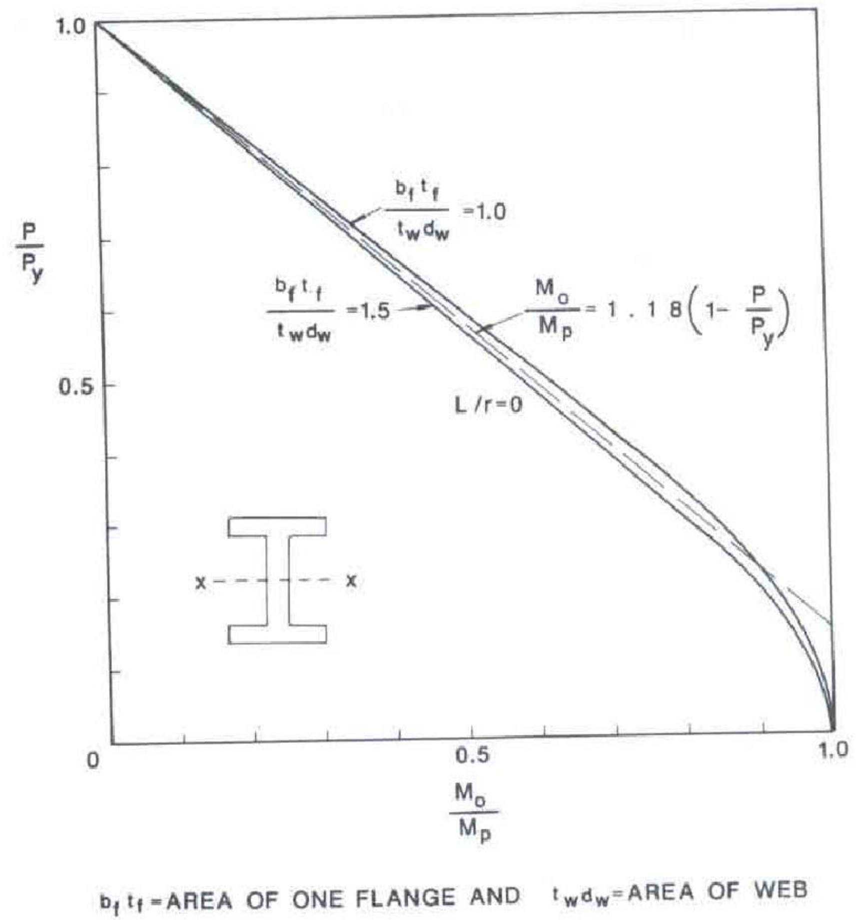


Fig. 4.5. Correlation between theory and test for an eccentrically loaded stub column. (*Proc. AISC Nat'l. Engr. Conf.*, 1956.)

Approximate Equation for Axial Effect

- **When slenderness, I.e. buckling doesn't govern**



What governs?

- How do you know if yielding governs or stability governs
- AISC code uses interaction equations for combined axial and bending
- Equations and approach varies between
  - AISC ASD
  - AISC ASD Plastic Design
  - AISC LRFD

AISC ASD

•Interaction Equations

If stability controls, the interaction equation is

$$\frac{f_a}{F_a} + \frac{f_{bx} C_{mx}}{F_{bx}(1 - f_a/F'_{ex})} + \frac{f_{by} C_{my}}{F_{by}(1 - f_a/F'_{ey})} \leqslant 1.0$$

If the yielding of material controls, the interaction equation is

$$\frac{f_a}{0.60F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leqslant 1.0 \qquad (3.10.10)$$

if $f_a/F_a \leqslant 0.15$

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leqslant 1.0$$

AISC "ASD" Plastic Design

NAVFAC

•Interaction Equations

The AISC/PD format provides two interaction equations for the design of beam-columns.

If yielding controls,

$$\frac{P}{P_y} + \frac{M}{1.18M_p} \leqslant 1.0 \qquad (3.10.12)$$

where

$P_y = A_g F_y$ = yield load of the section where A_g is the area of the cross section

$M_P = ZF_y$ = full plastic moment capacity of the section where Z is plastic-section modulus

•P and M are factored loads in this equation

AISC "ASD" Plastic Design

•Interaction Equation continued

If stability controls,

$$\frac{P}{P_u} + \frac{C_m M_0}{M_m(1 - P/P_{ek})} \leqslant 1.0 \qquad (3.10.13)$$

where

P_u = ultimate axial compressive strength of the axially loaded column taken as 1.7 times AISC/ASD column curve using the effective length of the column

M_m = maximum resisting moment in the absence of axial force, taken as M_p if the member is braced against lateral torsional buckling and taken as

$$M_m = \left[1.07 - \frac{(L/r_y)\sqrt{F_y}}{3160}\right] = M_p \leqslant M_d \qquad (3.10.14)$$

if the member fails by lateral torsional buckling.

In Eq. (3.10.14) the units are inches and ksi.

AISC LRFD

•Interaction Equation

The AISC/LRFD format based on the exact inelastic solutions of 82 beam-columns,[13] recommends the following interaction equations for sway and nonsway beam-columns.

For $P/\phi_c P_u \geqslant 0.2$

$$\frac{P}{\phi_c P_u} + \frac{8}{9}\left(\frac{M_{ax}}{\phi_b M_{ux}} + \frac{M_{ay}}{\phi_b M_{uy}}\right) \leqslant 1.0 \qquad (3.10.15)$$

For $P/\phi_c P_u < 0.2$

$$\frac{P}{2\phi_c P_u} + \frac{M_{ax}}{\phi_b M_{ux}} + \frac{M_{ay}}{\phi_b M_{uy}} \leqslant 1.0 \qquad (3.10.16)$$

AISC LRFD Alternate Interaction

In addition to Eqs. (3.10.15) and (3.10.16), the LRFD Specification also recommends a set of nonlinear interaction equations in its Appendix that are valid for nonsway members with end moments M_{ox} and M_{oy}. These equations are given as follows:

If yielding occurs,

$$\left(\frac{M_{ox}}{\phi_b M'_{px}}\right)^{\zeta} + \left(\frac{M_{oy}}{\phi_b M'_{py}}\right)^{\zeta} \leqslant 1.0 \tag{3.10.26}$$

If stability controls

$$\left(\frac{C_{mx} M_{ox}}{\phi_b M'_{nx}}\right)^{\eta} + \left(\frac{C_{my} M_{oy}}{\phi_b M'_{ny}}\right)^{\eta} \leqslant 1.0 \tag{3.10.27}$$

where

$$\zeta = 1.6 - \frac{P/P_y}{2[\ln (P/P_y)]} \quad \text{for} \tag{3.10.28}$$

$$\eta = \begin{cases} 0.4 + \dfrac{P}{P_y} + \dfrac{b_f}{d} \geqslant 1.0 & \text{for} \quad b_f/d \geqslant 0.3 \\ 1 & \text{for} \quad b_f/d < 0.3 \end{cases} \tag{3.10.29}$$

So how do you consider Axial Effects

- If axial load is very low then response is generally governed by yielding, I.e. flexure
  - Plastic moment can be reached and the magnitude of Mp is not effected significantly by axial load
- If axial load is moderate, and you are sure of adequate bracing is provided to avoid buckling (incl. LTB)
  - Plastic moment can be reached, but magnitude of Mp should be adjusted to account for axial load
- If axial load is moderate with inadequate bracing or axial load is high
  - Assume member fails when limit state is reached

Shear Effects

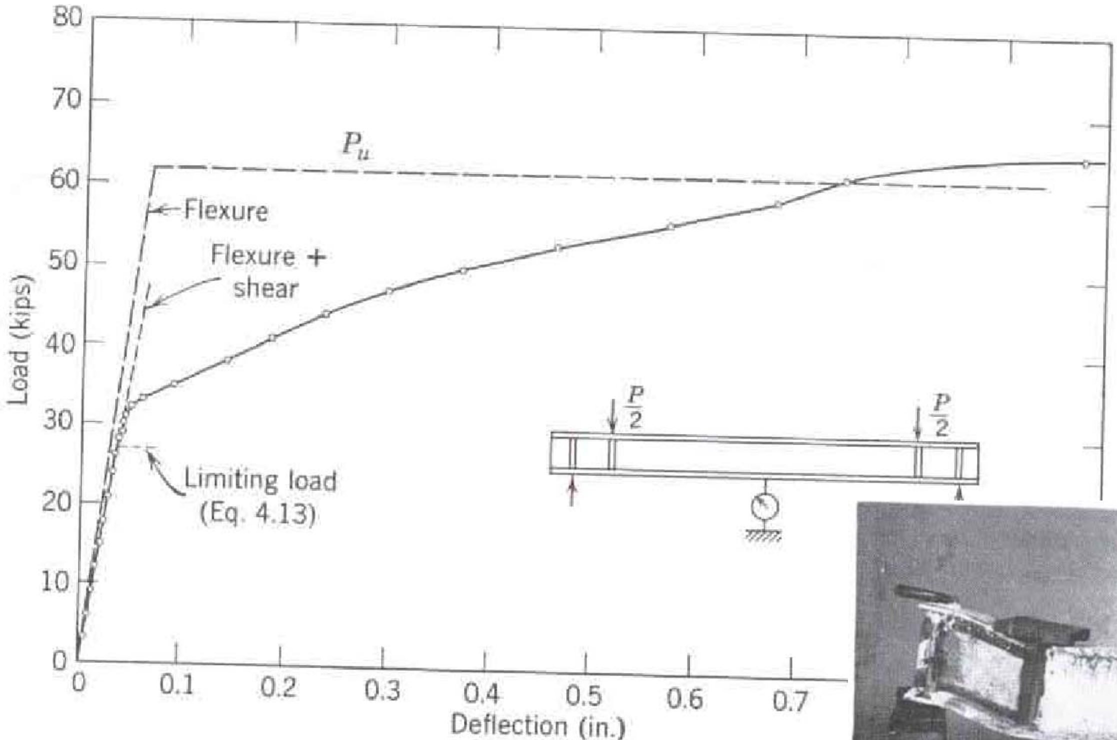


Fig. 4.9. Load–deflection curve of a beam subjected t

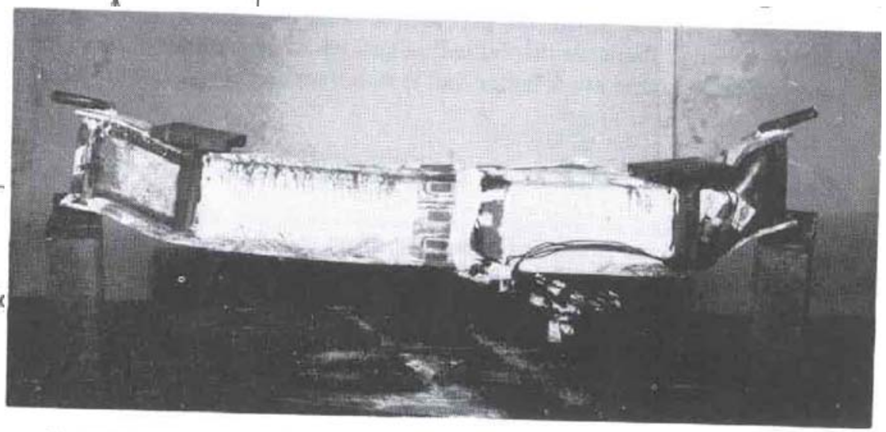


Fig. 4.10. Photograph of the beam whose load–deflection curve is shown in Fig. 4.9.

- **Generally, not an issue**

Local Buckling

- **AISC addresses through code provisions**

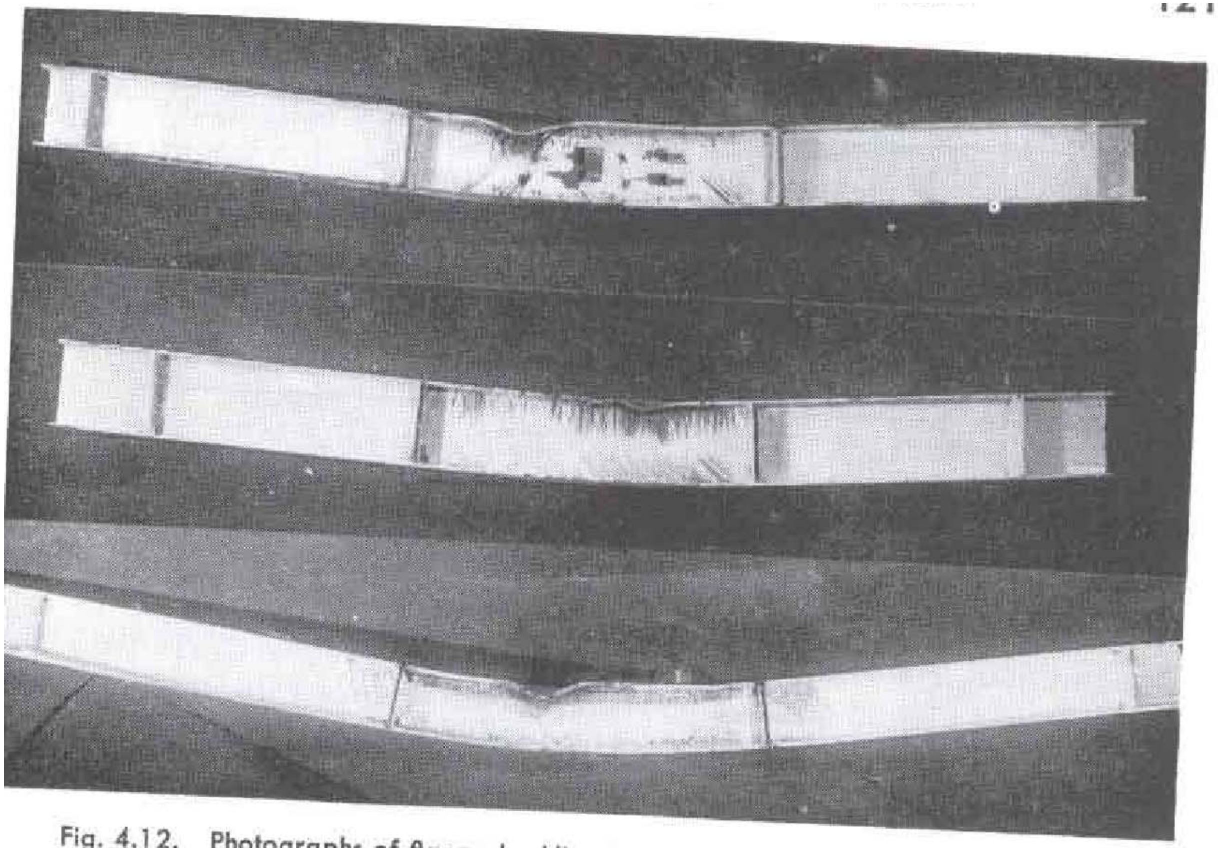


Fig. 4.12. Photographs of flange buckling in three beams with different b/t ratios.

Lateral Buckling

- AISC addresses through code provisions
- Not a big concern for most PC analysis, but should be aware of it

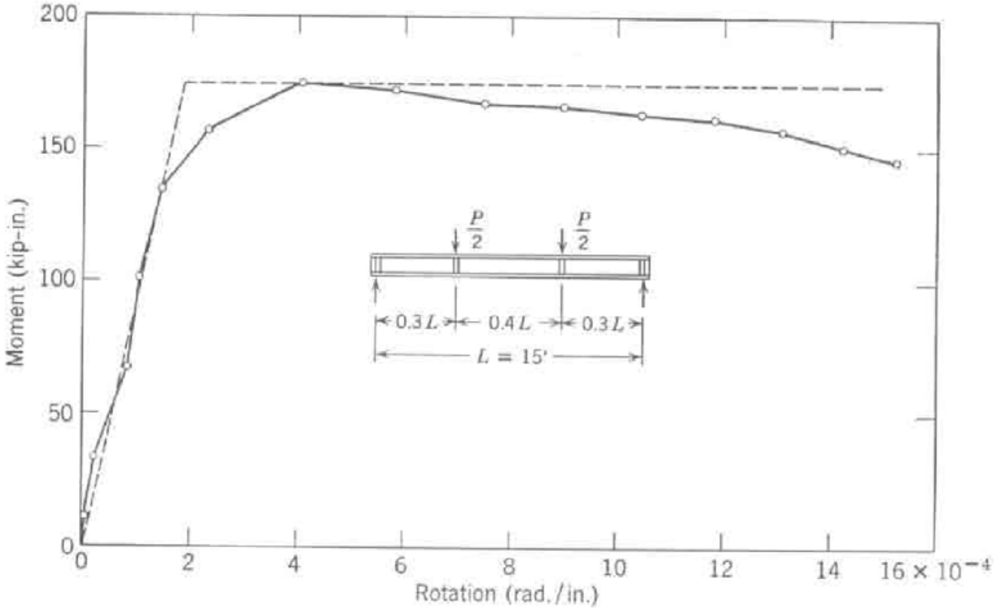


Fig. 4.15. Moment rotation curve showing the effect of lateral buckling.[1.16]

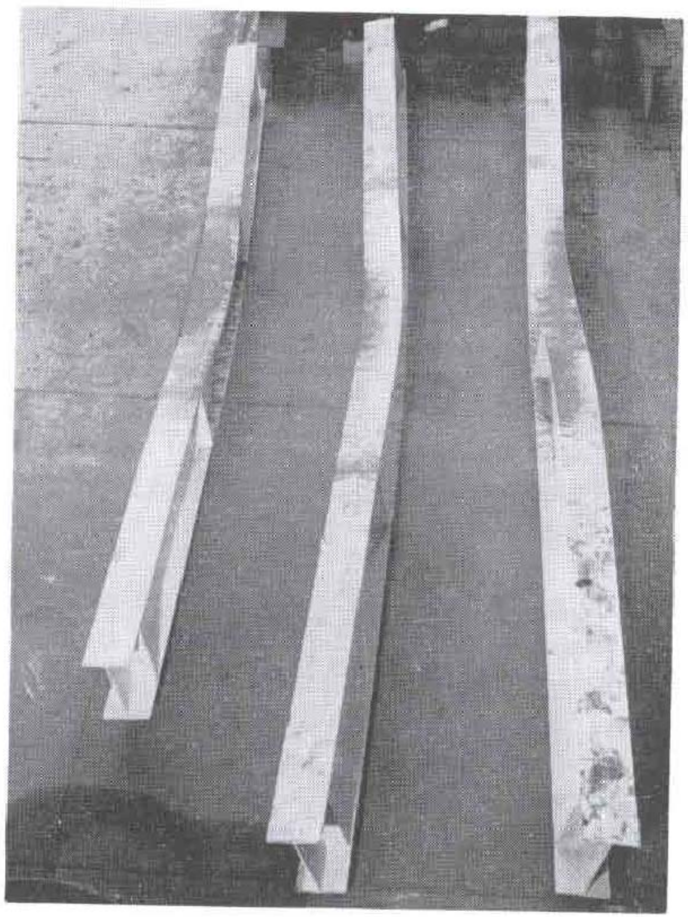


Fig. 4.16. Photograph of three beams that have buckled laterally.

Analysis Programs

- So can you expect general analysis programs to monitor and judge all of these limit states?
  - Generally, NO! Analysis programs don't generally check all limit states, this is done during "design code checks" for many applications
- If software uses a P-M-M hinge, you need to know how it was derived and how code handles stability
- Designer must be aware of and check all limit states to ensure the structure performs as the analysis assumes……

Where do you look for more guidance?

- Generally you can look at inelastic or plastic design references
  - Ex. Plastic Design of Steel Frames by Dr. Lynn Beddle or similar text by B.G. Neal
  - Look at plastic design requirements in material codes (AISC)
  - Look at more recent references on inelastic analysis and design

AISC Requirements for Plastic Design

- **AISC allows plastic design in accordance with Spec. A5.1**
  - **References B5.2, C1.1, C2.1a, C2.2a, E1.2, F1.3, H1 and I1**
- **B5.2 → Local Buckling, section must be compact (or maybe better)**
- **C1.1 → Determine Mu based on second-order plastic analysis**
- **C2.1a/C2.2a → Limits on magnitude of axial force for plastic design**

AISC Requirements for Design

- E1.2 → Limits on slenderness of columns when using plastic design
- F1.3 → Bracing requirements surrounding hinge locations
- H1 → Combined force and torsion
- I1 → Calculating strength of composite members in plastic design

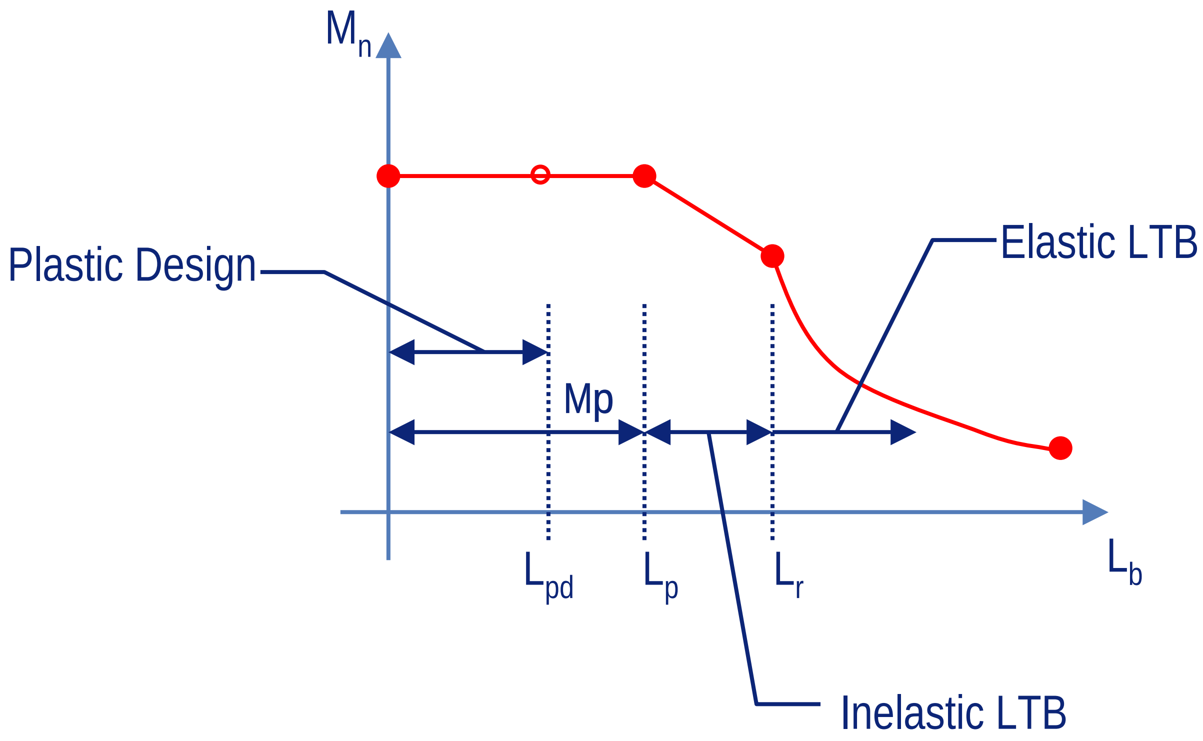
Do you stop there? NO, Look at commentary

Closer look at AISC provisions

- **Compact Section (LFRD) – capable of developing fully plastic stress distribution and possess inelastic ductility ratio of 3 before the onset of local buckling**
- **AISC Spec. Comm. B5 – greater inelastic rotation capacity than those provided by a compact section may be required in some structures. In order to develop a ductility factor of 5 to 15 in a member, it is prudent to provide for an inelastic rotation of 7 to 9 times the elastic rotation. To provide this rotation capacity a "seismic" section should be provided (Table C-B5.1 and AISC Seismic supplement)**
- **Ductility factor = (Total deformation at max load / elastic limit deformation)**
- **Rotation Capacity = Overall rotation at factored load state / idealized rotation corresponding to elastic theory applied at M= Mp**
- **Inelastic Ductility Ratio = ratio of strain at fracture to strain at yield**

Closer look at AISC Provisions

- **AISC Spec. Comm. F1.3 – Different bracing formula for higher rotation values**



Concrete - ACI Provisions

- Since members are where hinges are expected to form are undergoing significant inelastic deformation, look closely at seismic requirements (Chapter 21 of ACI 318) for “special” frames or systems
- Consider if “seismic” detailing is appropriate for areas of “ties” to ensure adequate confinement to develop tension tie
- Rotation capacity of concrete members is sensitive to reinforcement ratio and confinement

Concrete “Plastic Hinges”

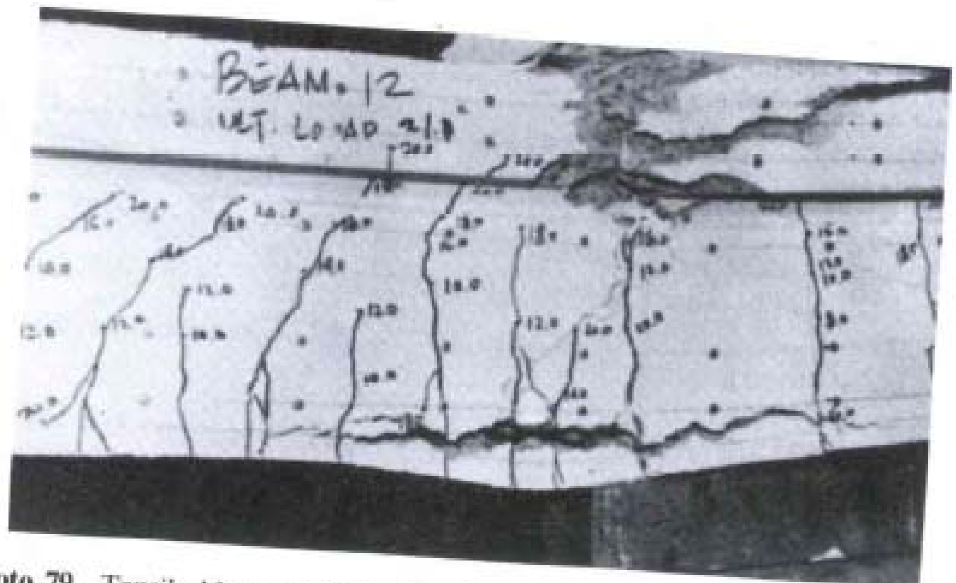


Photo 79 Tensile hinge at failure in a T-beam with rectangular confining reinforcement. (Nawy and Potyondy.)

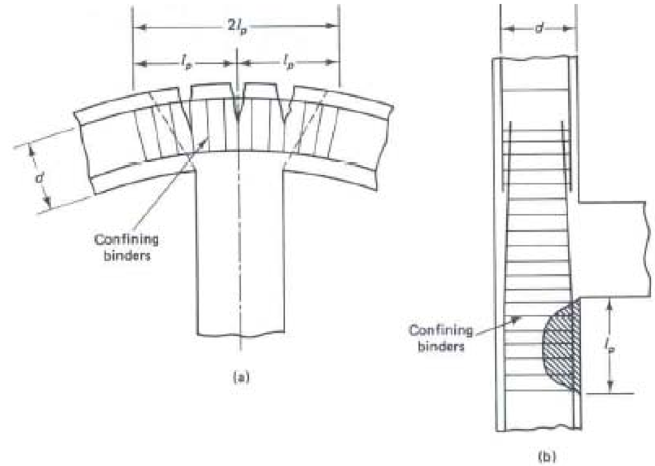


Figure 13.24 Plasticity zones l_p in plastic hinges: (a) tensile hinge; (b) compres-

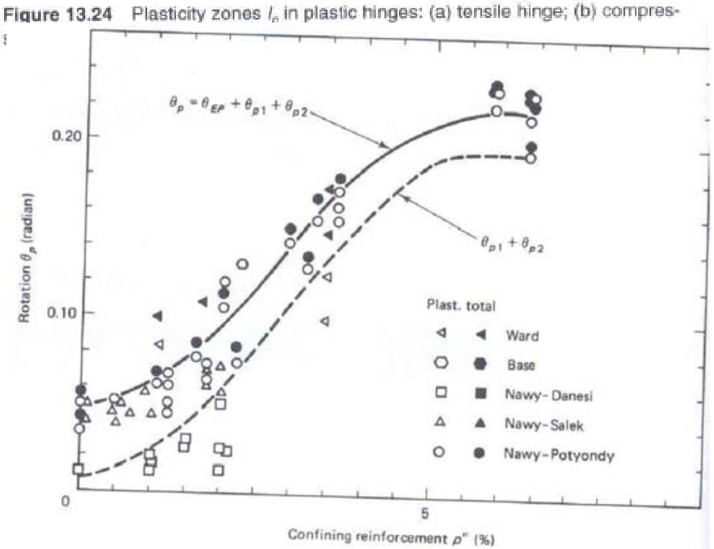


Figure 13.25 Comparison of plastic rotation with results of other authors.

Concrete – Detailing

- If you expect significant inelastic behavior – detail for it
- Look at ACI seismic requirements
- Look at DAHSCWE for recommendations for reinforcing detailing in regions of high rotations
- Look at DAHSCWE for discussion of membrane behavior
- Look at the suggested references in the UFC Appendix

Summary

- UFC can't address details of inelastic behavior of all materials or even all of the elastic material specific behavior
- Appendix to UFC attempts to point designers to references on some issues
- Engineer must still insure they understand the response of the structure they are modeling, the limits of the analysis technique they are employing, and meet material specific design requirements